

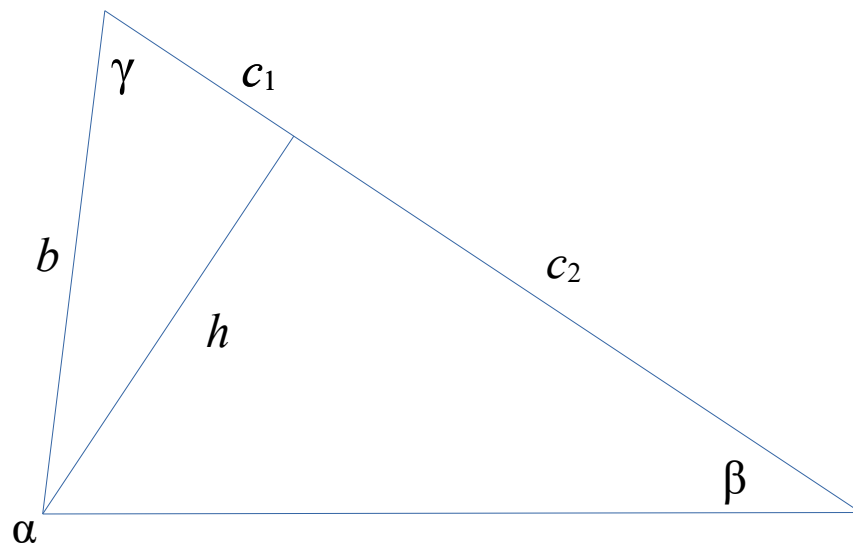
The Cosine Rule, a generalisation of Pythagoras' Theorem to any triangle

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The proof is based on the example demonstrated in <https://byjus.com/maths/law-of-cosines/>

Much more can be found at https://en.wikipedia.org/wiki/Law_of_cosines



$$c = c_1 + c_2.$$

$$c_1 = b \cos \gamma.$$

$$c_2 = c - b \cos \gamma.$$

$$h = b \sin \gamma.$$

$$a^2 = h^2 + c_2^2,$$

$$= (b \sin \gamma)^2 + (c - b \cos \gamma)^2,$$

$$= b^2 \sin^2 \gamma + c^2 - 2cb \cos \gamma + b^2 \cos^2 \gamma,$$

$$= b^2 (\sin^2 \gamma + \cos^2 \gamma) + c^2 - 2cb \cos \gamma.$$

But, by Pythagoras' Theorem on a triangle with unit hypotenuse, $1 = \sin^2 \gamma + \cos^2 \gamma$,

$$\therefore a^2 = b^2 + c^2 - 2cb \cos \gamma.$$